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Multiplication worksheets by 1

Multiplying numbers in columns requires lots of practice. Our grade 4 worksheets range in difficulty from 2 x 1 digits to 3 x 3 digits. These worksheets complement our grade 4 mental multiplication worksheets which focus on practicing "in your head" multiplication skills. Find all of our multiplication worksheets, from basic multiplication facts to multiplying multi-digit whole numbers in columns. Our multiplication worksheets start with the basic multiplication facts and progress to multiplying large numbers in columns. We emphasize "mental multiplication" exercises to improve numeracy skills. Grade 2 multiplication worksheets Grade 3 multiplication worksheets Grade 4 mental multiplication worksheets Grade 4 multiply in columns worksheets Grade 5 multiplication worksheets Grade 6 multiplication worksheets Multiplication facts: drills and practice Multi-digit multiplication: drills and practice Multiplication flashcards Topics include: Meaning of multiplication Arrays Multiplication Facts 2 & 3, 5, 10, 2-5 Multiplication Tables of 2, 5 & 10 Multiplication tables - missing factors Two times small numbers Two times multiples of 5 Two times whole tens Two times whole tens (missing factors) Multiplication word problems (within 25) Meaning of multiplication Multiplication sentences Multiply with arrays Multiply using a number line Multiplication facts (various practice up to 2-12) Multiplication tables Multiplication facts (missing factors) Multiplying 1-digit numbers by whole tens Multiplying 1-digit numbers by whole hundreds Multiply whole tens by whole tens Multiply whole tens (missing factors) Multiply in columns (1-digit by 2-4 digits) Multiplication word problems Multiplication tables 2-10, 2-12, random facts Multiplication tables 2-10, 2-12, missing factors Commutative property Distributive property Multiply 1-dit numbers by whole tens or hundreds Multiplying whole tens by whole tens (including missing factors) Multiply whole tens, whole hundreds and whole thousands Multiply 1-digit numbers by a number close to 100 Multiply in parts (1-digit by 2 or 3 digits) Mixed multiplication and division word problems Mixed 4 operations word problems Multiply in columns 1-digit by 2, 3 or 4 digits Multiply in columns 2-digits by 2, 3 or 4 digits Multiply in columns 3 digit by 3 digit Multiply by 10, 100 or 1,000 with missing factors Multiplying in parts (distributive property) Multiply 1 digit by 3 digit numbers mentally Multiply in columns up to 2x4 digits and 3x3 digits Mixed 4 operations word problems Distributive property Multiplying in columns up to 5 digit numbers Division worksheets Fractions worksheets Welcome to The Multiplying (1 to 12) by 1 (100 Questions) (A) Math Worksheet from the Multiplication Worksheets Page at Math-Drills.com. 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The Multiplying (1 to 12) by 1 (100 Questions) (A) Math Worksheet Page 1 The Multiplying (1 to 12) by 1 (100 Questions) (A) Math Worksheet Page 2 Other Versions: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z All More Multiplication Worksheets Welcome to the Math Salamanders Multiplication Printable Worksheets. Here you will find a wide range of free printable Multiplication Worksheets which will help your child improve their multiplying skills. Take a look at our times table worksheets, or check out our multiplication games or some multiplication word problems. This page contains links to other Math webpages where you will find a range of activities and resources. If you can't find what you are looking for, try searching the site using the Google search box at the top of each page. Once children have mastered place value to 100, and learn to count in steps of 2, 5 and 10, they are ready to start multiplication. Multiplication follows on naturally from counting in steps of different sizes. When children first learn multiplication, the learning is linked to addition which they are already very familiar with. So 2 + 2 + 2 + 2 becomes 2 four times or 2 x 4 (or 4 x 2). Once children have understood what multiplication is, they are then ready to start learning their tables, learning to multiply by one or two digit numbers, and then applying their knowledge to solve problems. At the very last stage in elementary math, they are ready to start multiplying decimals. The multiplication printable worksheets below will take your child through their multiplication learning step-by-step so that they are learning the math skills to move on to the next step, as well as starting off at a nice easy level to gain confidence. Here you will find our selection of free resources such as flashcards, multiplication charts and grids to help your child to learn their multiplication facts. Here you will find a selection of Multiplication Flashcards designed to help your child learn their Multiplication facts. Using flashcards is a great way to learn your Multiplication facts. They can be taken on a journey, played with in a game, or used in a spare five minutes daily until your child knows their Multiplication facts off by heart. Using these flashcards will help your child to: Learn their multiplication facts for the Multiplication tables. All the free Math flash cards in this section are informed by the Elementary Math Benchmarks for 2nd and 3rd Grade. Here you will find a selection of Multiplication Times Table Charts to 10x10 or 12x12 to support your child in learning their multiplication facts. There is a wide selection of multiplication charts including both color and black and white, smaller charts, filled charts and blank charts. Using these charts will help your child to: Learn their multiplication facts to 10x10 or 12x12; Practice their multiplication table. Negative Number Multiplication Here you will find a selection of Mental Multiplication sheets designed to help your child improve their mental recall of Multiplication Facts and learn their times tables. As your child progresses through the grades, they will learn their multiplication table to 5x5 and eventually to 10x10, or 12x12. Once they know their multiplication facts, they can start to learn related facts, e.g. if 3 x 4 = 12, then 30 x 4 = 120 and $300 x 4 = $1200. The multiplication printable worksheets below will support your child with their multiplication learning. Math Times Table Worksheets Top of Page These Multiplication Printable Worksheets below are designed to help your child improve their ability to multiply a range of numbers by multiples of 10 and 100 mentally. The following sheets develop children's ability to use and apply their tables knowledge to answer related questions. Using these sheets will help your child to: practice their multiplication table facts; multiply a range of numbers by 10 and 100; learn to use their multiplication tables to multiply by 10s and 100s. These Multiplication Printable Worksheets below involve children using their multiplication table facts to answer related questions involving decimals. Before your child tries written multiplication methods involving decimals, they should be confident using their multiplication table facts to multiply simple decimals mentally. Using these sheets will help your child to: consolidate and practice their multiplication table facts; use their multiplication tables to answer related facts involving decimals up to 2 decimal places (2dp). Here is our free generator for multiplication (and division) worksheets. This easy-to-use generator will create randomly generated multiplication worksheets for you to use. Each sheet comes complete with answers if required. The areas the generator covers includes: Multiplying with numbers to 5x5; Multiplying with numbers to 10x10; Multiplying with numbers to 12x12; Multiplying with 10s e.g. 4 x 30 Multiplying with 100s e.g. 6 x 400 Multiplying with tenths e.g. 3 x 0.7 Practicing a single times table; Practicing selected times tables; These generated sheets can be used in a number of ways to help your child with their times table learning. Here you will find a range of written multiplication printable worksheets. This is the first introduction of a written multiplication method at this stage. The following sheets develop children's ability to use a written method to multiply a multi-digit number by a single digit, starting off at a very basic level. Using these sheets will help your child to: learn and practice 2-digit multiplication. learn to multiply a multi-digit number by a single digit. Here you will find our 4th Grade Multiplication Printable Worksheet collection. These sheets are designed to help your child improve their ability to multiply a range of numbers by a two digit number. The following sheets will help your child to: multiply a range of 2 and 3 digit numbers by two digits. Our free 5th Grade Multiplication Printable Worksheet collection below is designed to help your child improve their ability to multiply a range of numbers involving decimals by a single digit. The following sheets will help your child to: multiply a range of decimals up to 2 decimal places (2dp) by a single digit; All the free Math sheets in this section are informed by the Elementary Math Benchmarks for 5th Grade. We have a range of Multiplication Word Problem worksheets. Each sheet comes in different levels of difficulty so that you can select an appropriate level for your child/class. Using our word problem sheets will help your child to: apply their math skills; select the correct multiplication fact needed to solve a problem; solve a range of problems, including 'real-life' problems. Here you will find a range of Free Printable Multiplication Games. The following games develop the Math skill of multiplying in a fun and motivating way. Using these sheets will help your child to: learn their multiplication facts; practice and improve their multiplication table recall; develop their strategic thinking skills. How to Print or Save these sheets Need help with printing or saving? Follow these 3 steps to get your worksheets printed perfectly! How to Print or Save these sheets Need help with printing or saving? Follow these 3 steps to get your worksheets printed perfectly! Sign up for our newsletter to get free math support delivered to your inbox each month. Plus, get a seasonal math grab pack included for free! The Math Salamanders hope you enjoy using these free printable Math worksheets and all our other Math games and resources. If you have any questions or need any information about our site, please get in touch with us using the 'Contact Us' tab at the top and bottom of every page. Multiplication is one of the basic math operations that we using in almost every aspect of our everyday living. You might notice it or you may not be too conscious about it but you are already multiplying things such as when you would buy something at the grocery or convenience store or when you would be computing your daily expenses at the end of the day. And with that, it is recommended that we master the art of multiplying numbers as early as in our childhood. This is to train our minds in multiplying varying values of numbers quickly and since it is one of the four basic math operations, it can be helpful in our day-to-day living. Multiplication Chart Worksheets Size: 118 KB Download Multiplication Worksheet Example tlsbooks.com Size: 177.8 KB Download Double Digit Multiplication Worksheet Example mathworksheetsland.com Size: 53.9 KB Download Multiplication Drills Worksheet Example commoncoresheets.com Size: 92.5 KB Download Multiplication Worksheets Example 3plearning.com Size: 2 MB Download Multiplication Fast Facts You might wonder why multiplication is often described as repeated addition. This is because if you would get the product of 4 x 4, it is equal to the sum of four fours which goes like this: 4 + 4 + 4 + 4. You may also see time worksheets examples in pdf. In order to determine whether an equation is a multiplication, you have to look at the sign used. Multiplication is actually symbolized in three ways: “x” as in 4 “x” 4 = 16 a centered dot (·) as in 4 · 4 = 16 writing the numbers next to each other with the use of parentheses () as in 4(4) = 16 or (4)(4) = 16 Keep in mind that the factor is the number that is being used as a multiplier in an equation, the multiplier is the one or two numbers that are being combined in the act of multiplying both in order to come up with a product, and the product is the result after multiplying two or more numbers. You may also check out expense worksheet examples in pdf. The answer to a multiplication problem or equation is called as the product and it can be derived by multiplying two multipliers. For examples, in the equation 4 x 4 = 16, the product here is 16 and the multipliers, which can also be called as the factors or terms, is 4 x 4. You might be interested in reading comprehension worksheet examples in pdf. When multiplying with decimal numbers, you might get overwhelmed but do not because all you have to do is just multiply the decimal fractions like natural numbers but just keep in mind to place the decimal point in the product according to the total number of decimal places of both multipliers. For example: 4.44 x 4.44 = 19.7136. Since there is a total of four decimal places from both of the multipliers, you have then to make sure that you have four decimal places on the product. You may also like resume worksheet examples in pdf. Ever encountered multiplication equation wherein the multipliers had varying signs? Do not worry; just multiply the numbers as if the signs do not exist. The rules go the product goes: If the two multipliers or factors has the same sign, give the product the similar sign. Hence, if it’s both negative, the product is also negative and if both are positive, then the product should also be positive. So, for example, 4 x 4 = 16 since both factors are positive, and -4 x -4 = -16 since both factors are negative. If the two multipliers have varying signs, the product should automatically use a negative sign. So, for example, 4 x -4 = -16. Quick Multiplication Drill Worksheet Example multiplication.com Size: 23.8 KB Download Two Digit Multiplication Worksheet Example triad.k12.oh.u Size: 45.1 KB Download Multiplication Squares Worksheet Example pasco.k12.fl.us Size: 1.5 MB Download Short Multiplication Worksheet Example tts-group.co.uk Size: 2.8 MB Download Interesting Facts about Multiplication Even if multiplication seems like the most ordinary thing in the world of mathematics, there are still some interesting facts about it. You may see some of it in the following: The word “multiply” is derived from the Latin term multus which means “multi” and the Latin term plex which means“fold”. You may also like self-assessment worksheet examples in pdf. Multiplication is just another method to repeatedly add numbers, so it is safe to call it “repeated addition”. You may also see writing worksheet examples in pdf. One of the properties in multiplication is the zero (0) property. The zero property means that whenever you multiply zero by any number (including zero), the answer is always zero. You may also see activity sheet examples & samples. Multiplication is actually a type of arithmetic and did you know that arithmetic is the oldest and the most basic branch of mathematics? You may also see sheet examples in DOC. Did you know that multiplication has an inverted version? It’s Division. How? Take this for example: 4 x 4 = 16. Now, if you would inverse it using division, it goes like this: 16 / 4 = 4. You may also check out coaching worksheet examples in pdf. When you would be multiplying an even number with the number, the answer or the product also ends up with an even number. Take this for example: 6 x 1 = 6, 6 x 2 = 12, 6 x 3 = 18, 6 x 4 = 24, 6 x 5 = 32, 6 x 6 = 36, 6 x 7 = 42, 6 x 8 = 48, 6 x 9 = 54, and 6 x 10 = 60 Did you know that the multipliers of an equation is previously referred to as multiplier and multiplicand? But now, the two is just referred to as multipliers and sometimes both called as factors. You may also check out math worksheets for students. The multiplication tables are also referred to as the “Table of Pythagoras.” It is coined in honor of Pythagoras of Samos, who is a famous Ionian Greek philosopher and mathematician. You may also like grammar worksheets. There was a clergyman way back in the 1500s who first used the symbol “?” for multiplication instead of what we know now as “x” symbol used for multiplying numbers. He was William Oughtred and he offered free math lessons during his time. However, Gottfried Wilhelm Leibniz objected this, thinking that it resembled the unknown “x”. You may also like biography worksheet examples. The Babylonians were the first to use multiplication tables and that happened over 4,000 years ago! You might be interested in goal setting worksheets. Quotes and Sayings Inspired by Multiplication We all know that multiplication is used in our daily lives, so do not be surprised to read the following quotations and famous sayings made by famous people that are inspired by the concept of multiplication. After all, mathematics is a part of our lives: “Civilization is the limitless multiplication of unnecessary necessities.” ? Mark Twain “Multiply your potentials with your plans and it will be equal to your purpose of existence. Your potentials are your seeds of greatness.” ? Israelmore Ayivor “In moments of crisis, all you gotta do is review your multiplication tables, and it’ll all blow over!” ? Andrea Camilleri “But I was thinking of a way To multiply by ten, And always, in the answer, get The question back again.” ? Lewis Carroll “Man falls from the pursuit of the ideal of plain living and high thinking the moment he wants to multiply his daily wants. Man’s happiness really lies in contentment.” ? Mahatma Gandhi “I am for a government rigorously frugal & simple, applying all the possible savings of the public revenue to the discharge of the national debt; and not for a multiplication of officers & salaries merely to make partisans, & for increasing, by every device, the public debt, on the principle of being a public blessing.” ? Thomas Jefferson “Thus you may multiply each stone 4 times & no more for they will then become oyles shining in ye dark and fit for magical uses. You may ferment them with gold and silver, by keeping the stone and metal in fusion together for a day, & then project upon metalls. This is the multiplication of ye stone in virtue. To multiply it in weight ad to it of ye first Gold whether philosophic or vulgar.” ? Isaac Newton “If it is a terrifying thought that life is at the mercy of the multiplication of these minute bodies [microbes], it is a consoling hope that Science will not always remain powerless before such enemies.” ? Louis Pasteur “A society’s competitive advantage will come not from how well its schools teach the multiplication and periodic tables, but from how well they stimulate imagination and creativity.” ? Albert Einstein Have fun solving these multiplication worksheets! You may also see Math Worksheets for Students. Add Tone Friendly Formal Casual Instructive Professional Empathetic Humorous Serious Optimistic Neutral 10 Examples of Public speaking 20 Examples of Gas lighting Arithmetical operation This article is about the mathematical operation. For other uses, see Multiplication (disambiguation). "·" redirects here. For the symbol, see Interpunct § In mathematics and science. This article needs additional citations for verification. Please help improve this article by adding citations to reliable sources. Unsourced material may be challenged and removed.Find sources: "Multiplication" – news · newspapers · books · scholar · JSTOR (April 2012) (Learn how and when to remove this message) Four bags with three marbles per bag gives twelve marbles (4 × 3 = 12). Multiplication can also be thought of as scaling. Here, 2 is being multiplied by 3 using scaling, giving 6 as a result. Multiplication is one of the four elementary mathematical operations of arithmetic, with the other ones being addition, subtraction, and division. The result of a multiplication operation is called a product. Multiplication is often denoted by the cross symbol, ×, by the mid-line dot operator, ·, by juxtaposition, or, on computers, by an asterisk, *. The multiplication of whole numbers may be thought of as repeated addition; that is, the multiplication of two numbers is equivalent to adding as many copies of one of them, the multiplicand, as the quantity of the other one, the multiplier; both numbers can be referred to as factors. This is to be distinguished from terms, which are added. a × b = b + ⋯ + b ⏟ a times . {\displaystyle a\times b=\underbrace {b+\cdots +b} _{a{\text{ times}}}.} Whether the first factor is the multiplier or the multiplicand may be ambiguous or depend upon context.[1] For example, the expression 3 × 4 {\displaystyle 3\times 4} , can be phrased as "3 times 4" and evaluated as 4 + 4 + 4 {\displaystyle 4+4+4} , where 3 is the multiplier, but also as "3 multiplied by 4", in which case 3 becomes the multiplicand. One of the main properties of multiplication is the commutative property, which states in this case that adding 3 copies of 4 gives the same result as adding 4 copies of 3. Thus, the designation of multiplier and multiplicand does not affect the result of the multiplication.[2] [3] Arithmetic operationsvte Addition (+) term + term summand + summand addend + addend augend + addend } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{term}}\,+\,{\text{term}}\\\scriptstyle {\text{summand}}\,+\,{\text{summand}}\\\scriptstyle {\text{addend}}\,+\,{\text{addend}}\\\scriptstyle {\text{augend}}\,+\,{\text{addend}}\end{matrix}}\right\}\,=\,} sum {\displaystyle \scriptstyle {\text{sum}}} Subtraction (−) term − term minuend − subtrahend } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{term}}\,-\,{\text{term}}\\\scriptstyle {\text{minuend}}\,-\,{\text{subtrahend}}\end{matrix}}\right\}\,=\,} difference {\displaystyle \scriptstyle {\text{difference}}} Multiplication (×) factor × factor multiplier × multiplicand } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{factor}}\,\times \,{\text{factor}}\\\scriptstyle {\text{multiplier}}\,\times \,{\text{multiplicand}}\end{matrix}}\right\}\,=\,} product {\displaystyle \scriptstyle {\text{product}}} Division (÷) dividend divisor numerator denominator } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\frac {\scriptstyle {\text{dividend}}}{\scriptstyle {\text{divisor}}}}\\[1ex]\scriptstyle {\frac {\scriptstyle {\text{numerator}}}{\scriptstyle {\text{denominator}}}}\end{matrix}}\right\}\,=\,} { fraction quotient ratio {\displaystyle \scriptstyle \left\{{\begin{matrix}\scriptstyle {\text{fraction}}\\\scriptstyle {\text{quotient}}\\\scriptstyle {\text{ratio}}\end{matrix}}\right.} Exponentiation (^) base exponent base power } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{base}}^{\text{exponent}}\\\scriptstyle {\text{base}}^{\text{power}}\end{matrix}}\right\}\,=\,} power {\displaystyle \scriptstyle {\text{power}}} nth root (√) radicand degree = {\displaystyle \scriptstyle {\sqrt[{\text{degree}}]{\scriptstyle {\text{radicand}}}}\,=\,} root {\displaystyle \scriptstyle {\text{root}}} Logarithm (log) log base (anti-logarithm) = {\displaystyle \scriptstyle \log _{\text{base}}({\text{anti-logarithm}})\,=\,} logarithm {\displaystyle \scriptstyle {\text{logarithm}}} vte Systematic generalizations of this basic definition define the multiplication of integers (including negative numbers), rational numbers (fractions), and real numbers. Multiplication can also be visualized as counting objects arranged in a rectangle (for whole numbers) or as finding the area of a rectangle whose sides have some given lengths. The area of a rectangle does not depend on which side is measured first—a consequence of the commutative property. The product of two measurements (or physical quantities) is a new type of measurement (or new quantity), usually with a derived unit of measurement. For example, multiplying the lengths (in meters or feet) of the two sides of a rectangle gives its area (in square meters or square feet). Such a product is the subject of dimensional analysis. The inverse operation of multiplication is division. For example, since 4 multiplied by 3 equals 12, 12 divided by 3 equals 4. Indeed, multiplication by 3, followed by division by 3, yields the original number. The division of a number other than 0 by itself equals 1. Several mathematical concepts expand upon the fundamental idea of multiplication. The product of a sequence, vector multiplication, complex numbers, and matrices are all examples where this can be seen. These more advanced constructs tend to affect the basic properties in their own ways, such as becoming noncommutative in matrices and some forms of vector multiplication or changing the sign of complex numbers. Main article: Multiplication sign See also: Multiplier (linguistics) In arithmetic, multiplication is often written using the multiplication sign (either × or × {\displaystyle \times }) between the factors (that is, in infix notation).[4] For example, 2 × 3 = 6 , {\displaystyle 2\times 3=6,} ("two times three equals six") 3 × 4 = 12 , {\displaystyle 3\times 4=12,} 2 × 3 × 5 = 6 × 5 = 30 , {\displaystyle 2\times 3\times 5=6\times 5=30,} 2 × 2 × 2 × 2 × 2 = 32. {\displaystyle 2\times 2\times 2\times 2\times 2=32.} There are other mathematical notations for multiplication: To reduce confusion between the multiplication sign × and the common variable x, multiplication is also denoted by dot signs, usually a middle-position dot (rarely period): 5 · 2 {\displaystyle 5\cdot 2} .[4] The middle dot notation or dot operator is now standard in the United States[4][5] and other countries.[6][clarification needed] When the dot operator character is not accessible, the interpunct (·) is used.[6] In most European and other countries that use a comma as a decimal point (and a period as a thousands separator), the multiplication sign or a middle dot is used to indicate multiplication. Historically, in the United Kingdom and Ireland, the middle dot was sometimes used for the decimal point to prevent it from disappearing in the ruled line, and the full stop (period) was used for multiplication. However, since the Ministry of Technology ruled in 1968 that the period be used as the decimal point,[7] and the International System of Units (SI) standard has since been widely adopted, this usage is now found only in the more traditional journals such as The Lancet.[8] In algebra, multiplication involving variables is often written as a juxtaposition (e.g., x y {\displaystyle xy} for x {\displaystyle x} times y {\displaystyle y} or 5 x {\displaystyle 5x} for five times x {\displaystyle x}), also called implied multiplication. The notation can also be used for quantities that are surrounded by parentheses (e.g., 5 (2) {\displaystyle 5(2)} , (5) 2 {\displaystyle (5)2} or (5) (2) {\displaystyle (5)(2)} for five times two). [9]This implicit usage of multiplication can cause ambiguity when the concatenated variables happen to match the name of another variable, when a variable name in front of a parenthesis can be confused with a function name, or in the correct determination of the order of operations.[10][11] In vector multiplication, there is a distinction between the cross and the dot symbols. The cross symbol generally denotes the taking a cross product of two vectors, yielding a vector as its result, while the dot denotes taking the dot product of two vectors, resulting in a scalar. In computer programming, the asterisk (as in 5*2) is still the most common notation. This is because most computers historically were limited to small character sets (such as ASCII and EBCDIC) that lacked a multiplication sign (such as · or ×),[citation needed] while the asterisk appeared on every keyboard.[12] This usage originated in the FORTRAN programming language.[13] The numbers to be multiplied are generally called the "factors" (as in factorization). The number to be multiplied is the "multiplicand", and the number by which it is multiplied is the "multiplier". Usually, the multiplier is placed first, and the multiplicand is placed second;[14][15] however, sometimes the first factor is considered the multiplicand and the second the multiplier. Also, as the result of multiplication does not depend on the order of the factors, the distinction between "multiplicand" and "multiplier" is useful only at a very elementary level and in some multiplication algorithms, such as the long multiplication. Therefore, in some sources, the term "multiplicand" is regarded as a synonym for "factor".[16] In algebra, a number that is the multiplier of a variable or expression (e.g., the 3 in 3 x y 2 {\displaystyle 3xy^{2}}) is called a coefficient. The result of a multiplication is called a product. When one factor is an integer, the product is a multiple of the other or of the product of the others. Thus, 2 × π {\displaystyle 2\times \pi } is a multiple of π {\displaystyle \pi } , as is 5133 × 486 × π {\displaystyle 5133\times 486\times \pi } . A product of integers is a multiple of each factor; for example, 15 is the product of 3 and 5 and is both a multiple of 3 and a multiple of 5. This section needs attention from an expert in mathematics. The specific problem is: defining multiplication is not straightforward and different proposals have been made over the centuries, with competing ideas (e.g. recursive vs. non-recursive definitions). See the talk page for details. WikiProject Mathematics may be able to help recruit an expert. (September 2023) The product of two numbers or the multiplication between two numbers can be defined for common special cases: natural numbers, integers, rational numbers, real numbers, complex numbers, and quaternions. 3 by 4 is 12. The product of two natural numbers r , s ∈ N {\displaystyle r,s\in \mathbb {N} } is defined as: r · s ≡ ∑ i = 1 s r = r + r + ⋯ + r ⏟ s times ≡ ∑ j = 1 r s = s + s + ⋯ + s ⏟ r times . {\displaystyle r\cdot s\equiv \sum _{i=1}^{s}r=\underbrace {r+r+\cdots +r} _{s{\text{ times}}}\equiv \sum _{j=1}^{r}s=\underbrace {s+s+\cdots +s} _{r{\text{ times}}}.} An integer can be either zero, a nonzero natural number, or minus a nonzero natural number. The product of zero and another integer is always zero. The product of two nonzero integers is determined by the product of their positive amounts, combined with the sign derived from the following rule: × + − + + − − − + (This rule is a consequence of the distributivity of multiplication over addition, and is not an additional rule.) In words: A positive number multiplied by a positive number is positive (product of natural numbers), A positive number multiplied by a negative number is negative, A negative number multiplied by a positive number is negative, A negative number multiplied by a negative number is positive. Two fractions can be multiplied by multiplying their numerators and denominators: z n · z ′ n ′ = z · z ′ n · n ′ , {\displaystyle {\frac {z}{n}}\cdot {\frac {z'}{n'}}={\frac {z\cdot z'}{n\cdot n'}},} which is defined when n , n ′ ≠ 0 {\displaystyle n,n'\neq 0} . There are several equivalent ways to define formally the real numbers; see Construction of the real numbers. The definition of multiplication is a part of all these definitions. A fundamental aspect of these definitions is that every real number can be approximated to any accuracy by rational numbers. A standard way for expressing this is that every real number is the least upper bound of a set of rational numbers. In particular, every positive real number is the least upper bound of the truncations of its infinite decimal representation; for example, π {\displaystyle \pi } is the least upper bound of { 3 , 3.1 , 3.14 , 3.141 , … } . {\displaystyle \{3,\;3.1,\;3.14,\;3.141,\ldots \}.} A fundamental property of real numbers is that rational approximations are compatible with arithmetic operations, and, in particular, with multiplication. This means that, if a and b are positive real numbers such that a = sup x ∈ A x {\displaystyle a=\sup _{x\in A}x} and b = sup y ∈ B y , {\displaystyle b=\sup _{y\in B}y,} then a · b = sup x ∈ A , y ∈ B x · y . {\displaystyle a\cdot b=\sup _{x\in A,y\in B}x\cdot y.} In particular, the product of two positive real numbers is the least upper bound of the term-by-term products of the sequences of their decimal representations. As changing the signs transforms least upper bounds into greatest lower bounds, the simplest way to deal with a multiplication involving one or two negative numbers, is to use the rule of signs described above in § Product of two integers. The construction of the real numbers through Cauchy sequences is often preferred in order to avoid consideration of the four possible sign configurations. Two complex numbers can be multiplied by the distributive law and the fact that i 2 = − 1 {\displaystyle i^{2}=-1} , as follows: (a + b i) · (c + d i) = a · c + a · d i + b i · c + b · d · i 2 = (a · c − b · d) + (a · d + b · c) i {\displaystyle {\begin{aligned}(a+b\,i)\cdot (c+d\,i)&=a\cdot c+a\cdot d\,i+b\,i\cdot c+b\cdot d\cdot i^{2}\\&=(a\cdot c-b\cdot d)+(a\cdot d+b\cdot c)\,i\end{aligned}}} A complex number in polar coordinates The geometric meaning of complex multiplication can be understood by rewriting complex numbers in polar coordinates: a + b i = r · (cos ⁡ (φ) + i sin ⁡ (φ)) = r · e i φ {\displaystyle a+b\,i=r\cdot (\cos(\varphi)+i\sin(\varphi))=r\cdot e^{i\varphi }} Furthermore, c + d i = s · (cos ⁡ (ψ) + i sin ⁡ (ψ)) = s · e i ψ , {\displaystyle c+d\,i=s\cdot (\cos(\psi)+i\sin(\psi))=s\cdot e^{i\psi },} from which one obtains (a · c − b · d) + (a · d + b · c) i = r · s · e i (φ + ψ) . {\displaystyle (a\cdot c-b\cdot d)+(a\cdot d+b\cdot c)i=r\cdot s\cdot e^{i(\varphi +\psi)}.} The geometric meaning is that the magnitudes are multiplied and the arguments are added. The product of two quaternions can be found in the article on quaternions. Note, in this case, that a · b {\displaystyle a\cdot b} and b · a {\displaystyle b\cdot a} are in general different. Main article: Multiplication algorithm The Educated Monkey—a tin toy dated 1918, used as a multiplication "calculator". For example: set the monkey's feet to 4 and 9, and get the product—36—in its hands. Many common methods for multiplying numbers using pencil and paper require a multiplication table of memorized or consulted products of small numbers (typically any two numbers from 0 to 9). However, one method, the peasant multiplication algorithm, does not. The example below illustrates "long multiplication" (the "standard algorithm", "grade-school multiplication"): 23958233 × 5830 ——————————————— 00000000 (= 23,958,233 × 0) 71874699 (= 23,958,233 × 30) 191665864 (= 23,958,233 × 800) + 119791165 (= 23,958,233 × 5,000) ——————————————— 139676498390 (= 139,676,498,390) In some countries such as Germany, the multiplication above is depicted similarly but with the original problem written on a single line and computation starting with the first digit of the multiplier:[17] 23958233 · 5830 ——————————————— 119791165 191665864 71874699 00000000 ——————————————— 139676498390 Multiplying numbers to more than a couple of decimal places by hand is tedious and error-prone. Common logarithms were invented to simplify such calculations, since adding logarithms is equivalent to multiplying. The slide rule allowed numbers to be quickly multiplied to about three places of accuracy. Beginning in the early 20th century, mechanical calculators, such as the Marchant, automated multiplication of up to 10-digit numbers. Modern electronic computers and calculators have greatly reduced the need for multiplication by hand. Methods of multiplication were documented in the writings of ancient Egyptian, Greek, Indian,[citation needed] and Chinese civilizations. The Ishango bone, dated to about 18,000 to 20,000 BC, may hint at a knowledge of multiplication in the Upper Paleolithic era in Central Africa, but this is speculative.[18][verification needed] Main article: Ancient Egyptian multiplication The Egyptian method of multiplication of integers and fractions, which is documented in the Rhind Mathematical Papyrus, was by successive additions and doubling. For instance, to find the product of 13 and 21 one had to double 21 three times, obtaining 2 × 21 = 42, 4 × 21 = 2 × 42 = 84, 8 × 21 = 2 × 84 = 168. The full product could then be found by adding the appropriate terms found in the doubling sequence:[19] 13 × 21 = (1 + 4 + 8) × 21 = (1 × 21) + (4 × 21) + (8 × 21) = 21 + 84 + 168 = 273. The Babylonians used a sexagesimal positional number system, analogous to the modern-day decimal system. Thus, Babylonian multiplication was very similar to modern decimal multiplication. Because of the relative difficulty of remembering 60 × 60 different products, Babylonian mathematicians employed multiplication tables. These tables consisted of a list of the first twenty multiples of a certain principal number n: n, 2n, ..., 20n; followed by the multiples of 10n: 30n 40n, and 50n. Then to compute any sexagesimal product, say 53n, one only needed to add 50n and 3n computed from the table.[citation needed] See also: Chinese multiplication table 38 × 76 = 2888 In the mathematical text Zhoubi Suanjing, dated prior to 300 BC, and the Nine Chapters on the Mathematical Art, multiplication calculations were written out in words, although the early Chinese mathematicians employed Rod calculus involving place value addition, subtraction, multiplication, and division. The Chinese were already using a decimal multiplication table by the end of the Warring States period.[20] Product of 45 and 256. Note the order of the numerals in 45 is reversed down the left column. The carry step of the multiplication can be performed at the final stage of the calculation (in bold), returning the final product of 45 × 256 = 11520. This is a variant of Lattice multiplication. The modern method of multiplication based on the Hindu–Arabic numeral system was first described by Brahmagupta. Brahmagupta gave rules for addition, subtraction, multiplication, and division. Henry Burchard Fine, then a professor of mathematics at Princeton University, wrote the following: The Indians are the inventors not only of the positional decimal system itself, but of most of the processes involved in elementary reckoning with the system. Addition and subtraction they performed quite as they are performed nowadays; multiplication they effected in many ways, ours among them, but division they did cumbrously.[21] These place value decimal arithmetic algorithms were introduced to Arab countries by Al Khwarizmi in the early 9th century and popularized in the Western world by Fibonacci in the 13th century.[22] Grid method multiplication, or the box method, is used in primary schools in England and Wales and in some areas[which?] of the United States to help teach an understanding of how multiple digit multiplication works. An example of multiplying 34 by 13 would be to lay the numbers out in a grid as follows: × 30 4 10 300 40 3 90 12 and then add the entries. Main article: Multiplication algorithm § Fast multiplication algorithms for large inputs The classical method of multiplying two n-digit numbers requires n2 digit multiplications. Multiplication algorithms have been designed that reduce the computation time considerably when multiplying large numbers. Methods based on the discrete Fourier transform reduce the computational complexity to O(n log n log log n). In 2016, the factor log log n was replaced by a function that increases much slower, though still not constant.[23] In March 2019, David Harvey and Joris van der Hoeven submitted a paper presenting an integer multiplication algorithm with a complexity of O (n log ⁡ n) . {\displaystyle O(n\log n).} [24] The algorithm, also based on the fast Fourier transform, is conjectured to be asymptotically optimal.[25] The algorithm is not practically useful, as it only becomes faster for multiplying extremely large numbers (having more than 2172912 bits).[26] Main article: Dimensional analysis One can only meaningfully add or subtract quantities of the same type, but quantities of different types can be multiplied or divided without problems. For example, four bags with three marbles each can be thought of as:[2] [4 bags] × [3 marbles per bag] = 12 marbles. When two measurements are multiplied together, the product is of a type depending on the types of measurements. The general theory is given by dimensional analysis. This analysis is routinely applied in physics, but it also has applications in finance and other applied fields. A common example in physics is the fact that multiplying speed by time gives distance. For example: 50 kilometers per hour × 3 hours = 150 kilometers. In this case, the hour units cancel out, leaving the product with only kilometer units. Other examples of multiplication involving units include: 2.5 meters × 4.5 meters = 11.25 square meters 11 meters/seconds × 9 seconds = 99 meters 4.5 residents per house × 20 houses = 90 residents Further information: Iterated binary operation § Notation The product of a sequence of factors can be written with the product symbol ∏ {\displaystyle \textstyle \prod } , which derives from the capital letter Π (pi) in the Greek alphabet (much like the same way the summation symbol ∑ {\displaystyle \textstyle \sum } is derived from the Greek letter Σ (sigma)).[27][28] The meaning of this notation is given by ∏ i = 1 4 (i + 1) = (1 + 1) (2 + 1) (3 + 1) (4 + 1) , {\displaystyle \prod _{i=1}^{4}(i+1)=(1+1)\,(2+1)\,(3+1)\,(4+1),} which results in ∏ i = 1 4 (i + 1) = 120. {\displaystyle \prod _{i=1}^{4}(i+1)=120.} In such a notation, the variable i represents a varying integer, called the multiplication index, that runs from the lower value 1 indicated in the subscript to the upper value 4 given by the superscript. The product is obtained by multiplying together all factors obtained by substituting the multiplication index for an integer between the lower and the upper values (the bounds included) in the expression that follows the product operator. More generally, the notation is defined as ∏ i = m n x i = x m · x m + 1 · x m + 2 · ⋯ · x n − 1 · x n , {\displaystyle \prod _{i=m}^{n}x_{i}=x_{m}\cdot x_{m+1}\cdot x_{m+2}\cdot \,\,\cdots \,\,\cdot x_{n-1}\cdot x_{n},} where m and n are integers or expressions that evaluate to integers. In the case where m = n, the value of the product is the same as that of the single factor xm; if m > n, the product is an empty product whose value is 1—regardless of the expression for the factors. By definition, ∏ i = 1 n x i = x 1 · x 2 · … · x n . {\displaystyle \prod _{i=1}^{n}x_{i}=x_{1}\cdot x_{2}\cdot \ldots \cdot x_{n}.} If all factors are identical, a product of n factors is equivalent to exponentiation: ∏ i = 1 n x = x · x · … · x = x n . {\displaystyle \prod _{i=1}^{n}x=x\cdot x\cdot \ldots \cdot x=x^{n}.} Associativity and commutativity of multiplication imply ∏ i = 1 n x i y i = (∏ i = 1 n x i) (∏ i = 1 n y i) {\displaystyle \prod _{i=1}^{n}{x_{i}y_{i}}=\left(\prod _{i=1}^{n}x_{i}\right)\left(\prod _{i=1}^{n}y_{i}\right)} and (∏ i = 1 n x i) a = ∏ i = 1 n x i a {\displaystyle \left(\prod _{i=1}^{n}x_{i}\right)^{a}=\prod _{i=1}^{n}x_{i}^{a}} if a is a non-negative integer, or if all x i {\displaystyle x_{i}} are positive real numbers, and ∏ i = 1 n x a i = x ∑ i = 1 n a i {\displaystyle \prod _{i=1}^{n}x^{a_{i}}=x^{\sum _{i=1}^{n}a_{i}}} if all a i {\displaystyle a_{i}} are non-negative integers, or if x is a positive real number. Main article: Infinite product One may also consider products of infinitely many factors; these are called infinite products. Notationally, this consists in replacing n above by the infinity symbol ∞. The product of such an infinite sequence is defined as the limit of the product of the first n factors, as n grows without bound. That is, ∏ i = m ∞ x i = lim n → ∞ ∏ i = m n x i . {\displaystyle \prod _{i=m}^{\infty }x_{i}=\lim _{n\to \infty }\prod _{i=m}^{n}x_{i}.} One can similarly replace m with negative infinity, and define: ∏ i = − ∞ ∞ x i = (lim m → − ∞ ∏ i = m 0 x i) · (lim n → ∞ ∏ i = 1 n x i) , {\displaystyle \prod _{i=-\infty }^{\infty }x_{i}=\left(\lim _{m\to -\infty }\prod _{i=m}^{0}x_{i}\right)\cdot \left(\lim _{n\to \infty }\prod _{i=1}^{n}x_{i}\right),} provided both limits exist.[citation needed] Main article: Exponentiation When multiplication is repeated, the resulting operation is known as exponentiation. For instance, the product of three factors of two (2×2×2) is "two raised to the third power", and is denoted by 23, a two with a superscript three. In this example, the number two is the base, and three is the exponent.[29] In general, the exponent (or superscript) indicates how many times the base appears in the expression, so that the expression a n = a × a × ⋯ × a ⏟ n = ∏ i = 1 n a {\displaystyle a^{n}=\underbrace {a\times a\times \cdots \times a} _{n}=\prod _{i=1}^{n}a} indicates that n copies of the base a are to be multiplied together. This notation can be used whenever multiplication is known to be power associative. Multiplication of numbers 0–10. Line labels = multiplicand. X axis = multiplier. Y axis = product.Extension of this pattern into other quadrants gives the reason why a negative number times a negative number yields a positive number.Note also how multiplication by zero causes a reduction in dimensionality, as does multiplication by a singular matrix where the determinant is 0. In this process, information is lost and cannot be regained. For real and complex numbers, which includes, for example, natural numbers, integers, and fractions, multiplication has certain properties: Commutative property The order in which two numbers are multiplied does not matter:[30][31] x · y = y · x . {\displaystyle x\cdot y=y\cdot x.} Associative property Expressions solely involving multiplication or addition are invariant with respect to the order of operations:[30][31] (x · y) · z = x · (y · z) . {\displaystyle (x\cdot y)\cdot z=x\cdot (y\cdot z).} Distributive property Holds with respect to multiplication over addition. This identity is of prime importance in simplifying algebraic expressions:[30][31] x · (y + z) = x · y + x · z . {\displaystyle x\cdot (y+z)=x\cdot y+x\cdot z.} Identity element The multiplicative identity is 1; anything multiplied by 1 is itself. This feature of 1 is known as the identity property:[30][31] x · 1 = x . {\displaystyle x\cdot 1=x.} Property of 0 Any number multiplied by 0 is 0. This is known as the zero property of multiplication:[30] x · 0 = 0. {\displaystyle x\cdot 0=0.} Negation −1 times any number is equal to the additive inverse of that number: (− 1) · x = (− x) {\displaystyle (-1)\cdot x=(-x)} , where (− x) + x = 0. {\displaystyle (-x)+x=0.} −1 times −1 is 1: (− 1) · (− 1) = 1. {\displaystyle (-1)\cdot (-1)=1.} Inverse element Every number x, except 0, has a multiplicative inverse, 1 x {\displaystyle {\frac {1}{x}}} , such that x · (1 x) = 1 {\displaystyle x\cdot \left({\frac {1}{x}}\right)=1} .[32] Order preservation Multiplication by a positive number preserves the order: For a > 0, if b > c, then ab > ac. Multiplication by a negative number reverses the order: For a < 0, if b > c, then ab < ac. The complex numbers do not have an ordering that is compatible with both addition and multiplication.[33] Other mathematical systems that include a multiplication operation may not have all these properties. For example, multiplication is not, in general, commutative for matrices and quaternions.[30] Hurwitz's theorem shows that for the hypercomplex numbers of dimension 8 or greater, including the octonions, sedenions, and trigintaduonions, multiplication is generally not associative.[34] Main article: Peano axioms In the book Arithmetices principia, nova methodo exposita, Giuseppe Peano proposed axioms for arithmetic based on his axioms for natural numbers. Peano arithmetic has two axioms for multiplication: x × 0 = 0 {\displaystyle x\times 0=0} x × S (y) = (x × y) + x {\displaystyle x\times S(y)=(x\times y)+x} Here S(y) represents the successor of y; i.e., the natural number that follows y. The various properties like associativity can be proved from these and the other axioms of Peano arithmetic, including induction. For instance, S(0), denoted by 1, is a multiplicative identity because x × 1 = x × S (0) = (x × 0) + x = 0 + x = x . {\displaystyle x\times 1=x\times S(0)=(x\times 0)+x=0+x=x.} The axioms for integers typically define them as equivalence classes of ordered pairs of natural numbers. The model is based on treating (x,y) as equivalent to x − y when x and y are treated as integers. Thus both (0,1) and (1,2) are equivalent to −1. The multiplication axiom for integers defined this way is (x p , x m) × (y p , y m) = (x p × y p + x m × y m , x p × y m + x m × y p) . {\displaystyle (x_{p},\,x_{m})\times (y_{p},\,y_{m})=(x_{p}\times y_{p}+x_{m}\times y_{m},\;x_{p}\times y_{m}+x_{m}\times y_{p}).} The rule that −1 × −1 = 1 can then be deduced from (0 , 1) × (0 , 1) = (0 × 0 + 1 × 1 , 0 × 1 + 1 × 0) = (1 , 0) . {\displaystyle (0,1)\times (0,1)=(0\times 0+1\times 1,\,0\times 1+1\times 0)=(1,0).} Multiplication is extended in a similar way to rational numbers and then to real numbers.[citation needed] The product of non-negative integers can be defined with set theory using cardinal numbers or the Peano axioms. See below how to extend this to multiplying arbitrary integers, and then arbitrary rational numbers. The product of real numbers is defined in terms of products of rational numbers; see construction of the real numbers.[35] There are many sets that, under the operation of multiplication, satisfy the axioms that define group structure. These axioms are closure, associativity, and the inclusion of an identity element and inverses. A simple example is the set of non-zero rational numbers. Here identity 1 is had, as opposed to groups under addition where the identity is typically 0. Note that with the rationals, zero must be excluded because, under multiplication, it does not have an inverse: there is no rational number that can be multiplied by zero to result in 1. In this example, an abelian group is had, but that is not always the case. To see this, consider the set of invertible square matrices of a given dimension over a given field. Here, it is straightforward to verify closure, associativity, and inclusion of identity (the identity matrix) and inverses. However, matrix multiplication is not commutative, which shows that this group is non-abelian. Another fact worth noticing is that the integers under multiplication do not form a group—even if zero is excluded. This is easily seen by the nonexistence of an inverse for all elements other than 1 and −1. Multiplication in group theory is typically notated either by a dot or by juxtaposition (the omission of an operation symbol between elements). So multiplying element a by element b could be notated as a · {\displaystyle \cdot } b or ab. When referring to a group via the indication of the set and operation, the dot is used. For example, our first example could be indicated by (Q / { 0 } , ·) {\displaystyle \left(\mathbb {Q} /\{0\},\,\cdot \right)} .[36] Numbers can count (3 apples), order (the 3rd apple), or measure (3.5 feet high); as the history of mathematics has progressed from counting on our fingers to modelling quantum mechanics, multiplication has been generalized to more complicated and abstract types of numbers, and to things that are not numbers (such as matrices) or do not look much like numbers (such as quaternions). Integers N × M {\displaystyle N\times M} is the sum of N copies of M when N and M are positive whole numbers. This gives the number of things in an array N wide and M high. Generalization to negative numbers can be done by N × (− M) = (− N) × M = − (N × M) {\displaystyle N\times (-M)=(-N)\times M=-(N\times M)} and (− N) × (− M) = N × M {\displaystyle (-N)\times (-M)=N\times M} The same sign rules apply to rational and real numbers. Rational numbers Generalization to fractions A B × C D {\displaystyle {\frac {A}{B}}\times {\frac {C}{D}}} is by multiplying the numerators and denominators, respectively: A B × C D = (A × C) (B × D) {\displaystyle {\frac {A}{B}}\times {\frac {C}{D}}={\frac {(A\times C)}{(B\times D)}}} . This gives the area of a rectangle A B {\displaystyle {\frac {A}{B}}} high and C D {\displaystyle {\frac {C}{D}}} wide, and is the same as the number of things in an array when the rational numbers happen to be whole numbers.[30] Real numbers Real numbers and their products can be defined in terms of sequences of rational numbers. Complex numbers Considering complex numbers z 1 {\displaystyle z_{1}} and z 2 {\displaystyle z_{2}} as ordered pairs of real numbers (a 1 , b 1) {\displaystyle (a_{1},b_{1})} and (a 2 , b 2) {\displaystyle (a_{2},b_{2})} , the product z 1 × z 2 {\displaystyle z_{1}\times z_{2}} is (a 1 × a 2 − b 1 × b 2 , a 1 × b 2 + a 2 × b 1) {\displaystyle (a_{1}\times a_{2}-b_{1}\times b_{2},a_{1}\times b_{2}+a_{2}\times b_{1})} . This is the same as for reals a 1 × a 2 {\displaystyle a_{1}\times a_{2}} when the imaginary parts b 1 {\displaystyle b_{1}} and b 2 {\displaystyle b_{2}} are zero. Equivalently, denoting − 1 {\displaystyle {\sqrt {-1}}} as i {\displaystyle i} , z 1 × z 2 = (a 1 + b 1 i) (a 2 + b 2 i) = (a 1 × a 2) + (a 1 × b 2 i) + (b 1 × a 2 i) + (b 1 × b 2 i 2) = (a 1 a 2 − b 1 b 2) + (a 1 b 2 + b 1 a 2) i . {\displaystyle z_{1}\times z_{2}=(a_{1}+b_{1}i)(a_{2}+b_{2}i)=(a_{1}\times a_{2})+(a_{1}\times b_{2}i)+(b_{1}\times a_{2}i)+(b_{1}\times b_{2}i^{2})=(a_{1}a_{2}-b_{1}b_{2})+(a_{1}b_{2}+b_{1}a_{2})i.} [30] Alternatively, in trigonometric form, if z 1 = r 1 (cos ⁡ ϕ 1 + i sin ⁡ ϕ 1) , z 2 = r 2 (cos ⁡ ϕ 2 + i sin ⁡ ϕ 2) {\displaystyle z_{1}=r_{1}(\cos \phi _{1}+i\sin \phi _{1}),z_{2}=r_{2}(\cos \phi _{2}+i\sin \phi _{2})} , then z 1 z 2 = r 1 r 2 (cos ⁡ (ϕ 1 + ϕ 2) + i sin ⁡ (ϕ 1 + ϕ 2)) . {\textstyle z_{1}z_{2}=r_{1}r_{2}(\cos(\phi _{1}+\phi _{2})+i\sin(\phi _{1}+\phi _{2})).} [30] Further generalizations See Multiplication in group theory, above, and multiplicative group, which for example includes matrix multiplication. A very general, and abstract, concept of multiplication is as the "multiplicatively denoted" (second) binary operation in a ring. An example of a ring that is not any of the number systems above is a polynomial ring (polynomials can be added and multiplied, but polynomials are not numbers in any usual sense). Division Often division, x y {\displaystyle {\frac {x}{y}}} , is the same as multiplication by an inverse, x (1 y) {\displaystyle x\left({\frac {1}{y}}\right)} . Multiplication for some types of "numbers" may have corresponding division, without inverses; in an integral domain x may have no inverse " 1 x {\displaystyle {\frac {1}{x}}} " but x y {\displaystyle {\frac {x}{y}}} may be defined. In a division ring there are inverses, but x y {\displaystyle {\frac {x}{y}}} may be ambiguous in non-commutative rings since x (1 y) {\displaystyle x\left({\frac {1}{y}}\right)} need not be the same as (1 y) x {\displaystyle \left({\frac {1}{y}}\right)x} .[citation needed] Dimensional analysis Multiplication algorithm Karatsuba algorithm, for large numbers Toom–Cook multiplication, for very large numbers Schönhage–Strassen algorithm, for huge numbers Multiplication table Binary multiplier, how computers multiply Booth's multiplication algorithm Floating-point arithmetic Multiply–accumulate operation Fused multiply–add Wallace tree Multiplicative inverse, reciprocal Factorial Genaille–Lucas rulers Lunar arithmetic Napier's bones Peasant multiplication Product (mathematics), for generalizations

Slide rule ^ {{cite journal |last=Foster |first=Colin |date=2022-03-01 |title=Getting multiplication the right way round |url= |journal=Mathematics in School |volume=51 |issue=2 |publisher=[[Mathematical association|The Mathematical Association (UK) |pages=16–17 |access-date=2025-05-15}} ^ a b Devlin, Keith (January 2011). "What Exactly is Multiplication?". Mathematical Association of America. Archived from the original on 2017-05-27. Retrieved 2017-05-14. With multiplication you have a multiplicand (written second) multiplied by a multiplier (written first) ^ Devlin, Keith (January 2011). "What exactly is multiplication?". profkeithdevlin.org. Archived from the original on 2024-12-12. Retrieved 2024-12-12. ^ a b c Musser, Gary L.; Peterson, Blake E.; Burger, William F. (2013). Mathematics for Elementary Teachers: A Contemporary Approach. John Wiley & Sons. p. 101. ISBN 978-1-118-48700-6. ^ Klose, Orval (1966). The Number Systems and Operations of Arithmetic. 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